XV. *Of the Temperament of thoſe muſical Inſtruments, in which the Tones, Keys, or Frets, are fixed, as in the Harpſichord, Organ, Guitar,* &c. *By Mr.* Tiberius Cavallo, *F. R. S.*

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THE ſcale of muſic, which is uſed at preſent, conſiſts of ſeven principal notes or ſounds, which muſicians denote by the letters of the alphabet A, B, C, D, E, F, and G; which, together with ſome intermediate ones, commonly called flats and ſharps, and the octave of the firſt, make 13 ſounds.

When thoſe ſounds are conſidered with reſpect to the firſt, they are called by the following names, *viz.* the prime or key-note, the ſecond minor, ſecond, third minor, third major, fourth, fourth major, fifth, ſixth minor, ſixth major, ſeventh minor, ſeventh major, and octave.

Muſical ſounds are produced by the vibrations of the ſonorous bodies, and they are acuter or graver as the vibrations performed in a given time are more or leſs in number; ſo that if a ſtring vibrating 100 times in a ſecond produces a certain ſound, and another ſtring vibrating 120 times in a ſecond produces another ſound, the latter is ſaid to be acuter, higher, or ſharper than the former.

The number of vibrations performed in a certain time principally depends on the thickneſs, length, and elaſticity of the ſonorous bodies; but as the ſimpleſt ſonorous bodies, and the fitteſt for examination, are thoſe ſtrings which are equal in every other reſpect, excepting in their lengths, becauſe the number of vibrations, which they perform in a given time, is ſimply in the proportion of their lengths, we ſhall conſider only thoſe in the preſent inveſtigation, the number of vibrations performed by other ſorts of ſonorous bodies being eaſily deduced from them.

As the above-mentioned 13 ſounds are all different from each other, the ſtrings which produce them differ in length, and of courſe in the number of the vibrations, which, when ſtruck, they perform in a certain time. Here follow the proportions which the numbers of vibrations performed in a given time, or the length of the ſtrings which expreſs thoſe 13 ſounds, bear to the firſt, prime, or key-note.

Firſt	1	Fourth	$\frac{3}{4}$	Seventh minor	$\frac{5}{9}$
Second minor	$\frac{15}{16}$	Fourth major	$\frac{32}{45}$	Seventh major	$\frac{8}{15}$
Second	$\frac{8}{9}$	Fifth	$\frac{2}{3}$	Octave	$\frac{1}{2}$
Third minor	$\frac{5}{6}$	Sixth minor	$\frac{5}{8}$		
Third major	$\frac{4}{5}$	Sixth major	$\frac{3}{5}$		

If, inſtead of many ſtrings having thoſe lengths in order to expreſs the 13 ſounds, or notes of an octave, one ſtring be divided according to thoſe proportions, and this ſtring be ſtopped conſecutively in the different points or diviſions; on being ſtruck, it will expreſs the correſponding ſounds. Thus, if a ſtring ſtretched between two fixed points, as CZ fig. 1. Tab. III. be ſtruck, it will produce a ſound called the prime, firſt or key-note; if it be ſtopped in the middle, one-half of the ſtring will ſound the octave, its length, compared to that of the whole ſtring, being in the proportion of 2 to 1; if two-thirds of the ſtring be cauſed to vibrate, the ſound produced will be the fifth, its length, compared to that of the whole ſtring, being as 2 to 3, and ſo of the reſt.

The higheſt ſound of the octave is expreſſed by the half of the ſtring; and if this half be divided again in the ſame manner or proportion, a higher octave will be obtained, the higheſt note of which will be expreſſed by a quarter of the original ſtring. This quarter may be divided again into a higher octave, and ſo on; therefore, a ſtring ſo divided may expreſs the ſounds of all the keys of an harpſichord or organ.

In regard to thoſe diviſions it muſt be obſerved, that as the notes of the ſecond octave bear the ſame proportion to the firſt note of that octave as the notes of the firſt octave reſpectively bear to the firſt note of that octave, or to the whole ſtring; and as the length of the ſtring expreſſing the firſt note of the ſecond octave, *viz.* *cZ*, fig. 1. is half the length of CZ, the firſt note of the firſt octave, it follows, that the length of the ſtring of every note in the ſecond octave is half the length of the correſponding note in the firſt octave; thus *g* in the ſecond octave is 120 inches long, and G in the firſt octave is 240 inches long, *viz.* twice 120. Hence, when the diviſions of the firſt octave are aſcertained, in order to find the diviſions of the notes of the ſecond octave, we need only take the half of the lengths expreſſing the notes in the firſt octave. By the very ſame reaſoning it is evident, that to find the diviſions for the third octave, we need only take the halves of the lengths which expreſs the notes of the ſecond octave, or the quarters of thoſe of the firſt octave, and ſo of the reſt.

The ſtring or line CZ, fig. 1. is divided in the above-mentioned manner, and in order to avoid confuſion, the diviſions of the principal notes only of the firſt and ſecond octave are annexed to it. The numbers under the line expreſs the lengths from Z to the diviſions to which they ſtand near. The letters juſt over the lines are the names of the notes or ſounds expreſſed by the correſponding lengths of the ſtring. The fractional numbers expreſs the proportion which each particular diviſion bears to the whole ſtring; and the Roman numbers denote the numerical names of each note with reſpect to its diſtance from the firſt, which is always included. For example, ſuppoſe the whole ſtring to be called C, and to be 360 inches long: then, if this ſtring be ſtopped in G, the part GZ will be 240 inches, *viz.* two-thirds of the whole ſtring CZ; the ſound expreſſed by it when ſtruck is called G, and it is the fifth note from C, which is the firſt or key-note. Again, if this ſtring be ſtopped in A, the part AZ will be equal to 216 inches, *viz.* three-fifths of the whole; the ſound produced by it is called A, and is a ſixth to the key-note C, &c.

It is evident, that if any of thoſe diviſions be conſidered as the firſt or key-note, then the other notes, though they retain their alphabetical names, muſt have their numerical names altered accordingly: for example, if we take D for the key-note, then A will be the fifth of it, whereas A was the ſixth when C was conſidered as the key-note; thus alſo B is the third of G, and the ſeventh of C, and ſo on.

Thus much having been premiſed, we may proceed to ſhew the meaning of what is called *the temperament* in a ſyſtem of muſical ſounds, and likewiſe the neceſſity of it. For this purpoſe it is neceſſary to recollect, firſt, that the ſtring, divided in the above-mentioned manner, exhibits the various notes or ſounds of the keys of an harpſichord, the pipes of an organ, &c. Secondly, that thoſe diviſions remain unalterable, ſo that the harpſichord, when tuned, cannot be altered in the courſe of performing on it; and, thirdly, that when any one of thoſe notes or diviſions is conſidered as the key-note, its ſecond, third, fourth, fifth, &c. muſt bear their reſpective proportions, according to what has been ſaid in the preceding pages.

Now, if amongſt the diviſions of the ſtring CZ, fig. 1. we take D for the firſt or key-note, its length being 320 inches, the length of its fifth muſt be 213⅓ inches, *viz.* two-thirds of 320, that being the proportion which the fifth muſt bear to the key-note; but amongſt the diviſions of the ſtring, there is none equal to 213⅓ inches; therefore, there is not a note among them which may ſerve for a fifth to D: however, as the length of AZ, *viz.* 216, is the neareſt to 213⅓, this A muſt be taken for the fifth of D. It is evident, that this is an imperfect fifth of D; but if, in order to render it perfect, we make AZ equal to 213⅓ inches inſtead of 216, then it will be a redundant ſixth to C, when C is conſidered as the key-note; the beſt expedient, therefore, is to divide the imperfection between the two lengths, *viz.* to make AZ neither ſo long as 216, nor ſo ſhort as 213⅓, which will render the diſagreeable ſenſation, ariſing from the improper length, the leaſt poſſible. This alteration of the juſt lengths of ſtrings, neceſſary for adapting them to ſeveral key-notes, is called *the temperament:* and the beſt temperament in a ſet of muſical ſounds is evidently ſuch a partition of the natural imperfections, as will render all the chords equally and the leaſt diſagreeable poſſible.

What has been exemplified in D and A may be ſaid of all the other notes; ſo that if any one of them be a perfect third, fifth, &c. with reſpect to one key-note, it will be found to be imperfect with reſpect to others. Hence it is manifeſt, firſt, that in a ſet of muſical keys, pipes, or frets, a temperament is abſolutely neceſſary; and, ſecondly, that the harpſichord, organ, guitar, or any other inſtrument in which the notes are fixed, ſo as not to be alterable by the performer's hands, muſt be imperfect even when tuned in the beſt manner poſſible; for by the temperament we can divide, but not annihilate, the imperfection.

Other inſtruments, in which the notes are not fixed, as the violin, violoncello, &c. are perfect, becauſe the performer ſtops the ſtrings upon them in different places, even for ſounding the notes of the ſame name. Thus a ſkilful performer, in order to ſound A, will ſtop the ſtring a little farther from the bridge when he plays in the key of C, *viz.* when C is conſidered as the key-note, than when he plays in the key of D.

Moſt people imagine, that the ſcale of muſic is capable of many different temperaments; and, agreeable to this ſuppoſition, the writers on harmonics have propoſed different temperaments; but it will be ſhewn in the ſequel, that the nature of the ſcale admits of only one temperament capable of rendering the imperfection and the harmony equal throughout; and that it is impoſſible to form a different and more advantageous ſcale.

Before we begin with the inveſtigation of this ſubject, it will be neceſſary to explain certain principles, the want of which may poſſibly raiſe ſome doubts in the minds of thoſe perſons who are not much acquainted with the theory of muſical ſounds. In the firſt place it muſt be obſerved, that the proportion of 2 to 3 for the fifth, the proportion of 1 to 2 for the octave, and in ſhort the proportions of all the notes, are not aſſumed at pleaſure; but they have been determined from conſtant experience, *viz.* from the agreeable or diſagreeable effects produced when two different notes are ſounded at the ſame time.

To render this more evident, let two ſtrings equal in every reſpect be ſtruck at the ſame time, and they will expreſs the ſame ſound preciſely, ſo that no ear can perceive any difference between them, and it is almoſt impoſſible to diſtinguiſh whether the ſound ariſes from two ſtrings, or from one only, excepting from the loudneſs. But if one of thoſe ſtrings be ſucceſſively ſtopped in different parts of its length, whilſt the other remains open as before, and if at every time they be both ſtruck together, their combined ſounds will be found to produce different effects, *viz.* ſometimes more or leſs pleaſing, and at other times more or leſs diſagreeable. When the combinations of the two ſounds are agreeable, they are called *concords*; and when diſagreeable, they are called *diſcords*.

Experience evinces, that the beſt concord is when the length of one ſtring is to the length of the other as 1 to 2, every other circumſtance being the ſame in both. This proportion forms the octave. The next beſt concord is the fifth, *viz.* when the lengths of the two ſtrings are as 2 to 3, after which come the proportions of 3 to 4, 4 to 5, 3 to 5, 5 to 6, and 5 to 8, for the other concords. The other proportions beſides theſe are diſagreeable in a greater or leſs degree, unleſs they are greater than the proportion of 1 to 2; but in that caſe it will be found, that the proportions which produce agreeable combinations are the double, quadruple, octuple, &c. of thoſe mentioned above, *viz.* are their octaves, double octaves, &c.: thus the proportion of 1 to 4 produces a very agreeable concord, becauſe 1 to 4 is the double of 1 to 2, *viz.* it expreſſes a double octave.

Secondly, it appears, from the foregoing obſervations, that if we have the length of a ſtring, or the proportion of a note in any part of the ſtring, we may eaſily find its octaves by taking its double, or its half, or the double of the double, &c.: for inſtance, in fig. 1. if *c*Z be given equal to 90 inches, we may find its octave below by taking twice 90, *viz.* 180, or the octave of this octave, which is 360, *viz.* equal to twice 180, or to four times 90; and, on the other ſide, we may find the octave above of the given note by taking its half, which is 45, &c.

It is now neceſſary to ſhew why within the octave there are admitted only thirteen different notes, *viz.* eight principal ones, and five others, called ſharps and flats.

The line XY, fig. 2. repreſents a muſical ſtring, the length of which is ſuppoſed to be divided into 13286025 equal parts *. On one ſide of this line there are the diviſions of ſeven ſucceſſive octaves, *viz.* the half of XZ, a quarter of it, &c.; and on the other ſide are the diviſions of a ſeries of fifths, *viz.* the fifth of the whole ſtring, the fifth of this fifth, and ſo on, which are found by taking two-thirds of the whole ſtring, then two-thirds of thoſe two thirds, and ſo on.

Here we take notice only of the octaves and fifths, becauſe they are the principal and the beſt concords; ſo that a temperament being required, it is neceſſary firſt to take care, that theſe concords be not rendered inſufferable to the ear, the reſt admitting of a greater latitude in the temperament or deviation from the perfect ſtate. Beſides, it will appear in the ſequel, that all the other notes are derived from the ſeries of ſucceſſive fifths.

In whatever key a piece of muſic is performed, its fifth is the moſt predominant of its concords; and as the notes of muſic muſt be ſo ordered as that, for the ſake of modulation, any note may be conſidered as the key-note; therefore having found the fifth of the whole ſtring by taking two-thirds of its length, which gives a note called G, we muſt ſuppoſe, that this G may be conſidered as the key-note, conſequently muſt find its fifth, which gives D, as ſhewn in the figure, and ſo on, until we find one of thoſe ſucceſſive fifths, which coincides with one of the ſucceſſive octaves; for after that, to find more ſucceſſive fifths would be only repeating the ſame thing over again.

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* This number has been choſen, becauſe the line may in that caſe be divided into the neceſſary number of ſucceſſive octaves and fifths, without any common fractions, which renders the operation more eaſy and more perſpicuous; otherwiſe there might have been aſſumed any other number.

Indeed, if we carry the ſucceſſion of octaves and of fifths indefinitely far, we ſhall find, that no one of the fifths ever coincides perfectly with one of the octaves, and therefore the diviſion would have no end. However, as the length of the ſeventh octave comes ſo very near to the twelfth fifth, we muſt be contented with taking this ſeventh octave for the fifth of F, the difference between this note and the perfect fifth of F being about the hundredth part of its length; whereas, if we carry on the ſucceſſion of fifths and of octaves, we ſhall find, that amongſt thirty and more fifths none comes nearer to one of the octaves than the above-mentioned one, as may be ſeen in the following table, which contains a ſeries of ſucceſſive octaves, and another ſeries of ſucceſſive fifths, as in fig. 2.; but in the table the diviſions are carried farther on than in the figure:

Octaves.	Fifths.
13286025	13286025.
6643012,5	8857350.
3321506,25	5904900.
1660753,125	3936600.
830376,5625	2624400.

Octaves.	Fifths.
415188,28125	1749600.
207594,140625	1166400.
☞ 103797,0703125	777600.
51898,5351 +	518400.
25949,2675 +	345600.
12974,6387 +	230400.
6487,3193 +	153600.
3243,654 +	☞ 102400.
1621,827 +	68266,6 +
810,913 +	45511 +
405,456 +	30340,7 +
202,728 +	20227 +
101,364 +	6742,3 +
50,662 +	4494,8 +
25,331 +	2996,6 +
12,665 +	1997,7 +
	1331,8 +
	887,8 +
	592 +
	394,6 +
	263 +
	175,4 +
	117 +
	78 +
	52 +
	34,6 +
	23 +
	15,3 +

The number of fifths then in this ſeries is twelve, and as, from what has been ſaid above, when the diviſion expreſſing a certain note has been aſſigned in any part of a ſtring, we may eaſily find all its octaves above and below, it follows, that by finding all the octaves of thoſe twelve diviſions we ſhall have twelve diſtinct notes within half the ſtring, *viz.* within the firſt octave of the whole ſtring; to which, if the ſound of the whole ſtring be added, we ſhall have thirteen different ſounds, which ſhews why an octave comprehends neither more nor leſs than thirteen notes.

Without dwelling any longer upon the names or number of thoſe notes, I ſhall immediately proceed to find out the temperament.

It has been ſhewn above, and it is expreſſed in fig. 2. that the length of the ſtring for the laſt fifth is ſhorter than the length of the laſt octave, and alſo that one of them muſt be neceſſarily taken for both purpoſes; but here we muſt conſult nature, examining by the ear which of the two is leaſt diſagreeable. This, however, is ſoon decided; for imperfect octaves are quite inſufferable, whereas a certain degree of imperfection in the fifths is tolerable; therefore we are neceſſitated to leave the octaves perfect, and to let the ſeventh octave ſerve for the fifth of F. In this caſe it is evident, that each of the notes in the ſucceſſion of fifths is a perfect fifth to its preceding note, excepting the laſt, which would be by much too flat, and therefore it is neceſſary to divide the imperfection equally among them all.

For this purpoſe it muſt be conſidered, that as the twelve ſucceſſive fifths, together with the whole ſtring or firſt note, are each two-thirds of its preceding note; they form a geometrical ſeries, the ratio of which is $\frac{2}{3}$, its extremes are 13286025 and 102400, and the number of terms is 13. But becauſe inſtead of 102400, which is the laſt fifth, we muſt take the number 103797,0703125 (*viz.* the length of the laſt octave) for the laſt term of the ſeries; therefore the problem is reduced to the finding out of eleven mean proportionals between the two numbers 13286025 and 103797,0703125.

It is demonſtrated in almoſt every treatiſe on algebra and arithmetic, that in a geometrical progreſſion, as the above-mentioned one, the firſt or ſmalleſt extreme is to the laſt or greateſt extreme as unity is to a power of the ratio, the index of which is equal to the number of terms leſs one. Hence, in our caſe, in which the number of terms, including the two extremes, is 13, we ſhall have 103797,0703125 : 13286025 :: 1 : R^{12}, from which the ratio is found by dividing the ſecond number by the firſt, and extracting the twelfth root from the quotient, *viz.* $\frac{13286025}{103797,0703125} = 128$; and $\overline{128}\vert^{\frac{1}{12}} = 1,4983069$, which is the ratio ſought.

The ratio having been aſcertained, the ſucceſſion of tempered fifths is thus eaſily determined; *viz.* divide the length of the whole ſtring by this ratio, and the quotient gives the firſt tempered fifth; divide this fifth by the ſame ratio, and the quotient gives the ſecond tempered fifth; divide this ſecond fifth by the ſame ratio, and ſo on till the laſt fifth, which comes out equal to 103797, 21735 (ſee fig. 3.) which is ſo nearly equal to the length of the ſeventh octave, that the difference is truly inſignificant; but, if greater accuracy were required, we need only extract the proper root of 128 to a greater number of decimals.

Fig. 3. ſhews the diviſions of the ſtring XZ tempered in the above-mentioned manner; *viz.* the ſucceſſive fifths have been aſcertained firſt, and then, by taking their octaves, the whole ſet of diviſions has been completed. By comparing this figure with fig. 2. one may eaſily perceive, how ſmall is

the difference between the perfect fifths of the latter, and the tempered ones of the former.

The diviſions, thus aſcertained, form a ſeries of notes, in which the octaves only are perfect; but all the fifths, all the thirds, and in ſhort all the chords of the ſame denomination, are equally tempered throughout: ſo that whichever of them is taken for the key-note, its fifth, ſixth, &c. will have always the ſame proportion to it, and conſequently will always produce the ſame harmony when ſounded with it.

It is evident, that, beſides this, there can be no other temperament capable of producing equal harmony; for when the extremes of a geometrical ſeries and number of mean proportionals are given, there can be but one ſet of thoſe means: thus, if we are to find two mean proportionals between the numbers 2 and 16, theſe are neceſſarily 4 and 8; nor is it poſſible to aſſign any others.

If, on the other hand, we endeavour to find a better temperament by introducing more than thirteen notes within the limits of an octave, we ſhall find it impracticable, becauſe it has been ſhewn, in the preceding pages, that after the number thirteen, if the ſucceſſion of fifths be carried farther on, they will recede more from a coincidence with any one of the octaves.

This explanation of the nature, origin, and neceſſity of the temperament has been thought neceſſary for the ſake of perſpicuity; but the ſame end may be obtained by the following eaſier method. As the thirteen notes of an octave muſt be arranged ſo, that whichever of them be taken for the firſt or key-note, the ſecond, third, fourth, &c. may bear the ſame conſtant proportion to it; therefore it follows, that they muſt be in a geometrical proportion one of the other, ſo as to form a ſeries of thirteen numbers, the extremes of which are the whole ſtring and its half, *viz.* any number and its half. The ratio of this ſeries is found in the ſame manner as in the other ſeries, *viz.* the greateſt extreme is divided by the leaſt, and the twelfth root of the quotient is the ratio ſought. But the extremes are any aſſumed number and its half: and as the quotient of a number divided by the half of the ſame number is always equal to two; therefore, whatever be the length of the ſtring, the ratio is always $\overline{2}|^{\frac{1}{12}} = 1,0594+$, and if the length of the whole ſtring be divided by this ratio, *viz.* 1,0594+, the quotient will be the length of the ſtring expreſſing the ſecond note, which, divided by the ſame ratio, gives the third note, and ſo on; or elſe, inſtead of dividing the length of the whole ſtring by the ratio, you may multiply the half of it by the ratio, the product of which will give the ſeventh note, which multiplied by the ſame ratio gives the ſixth, and ſo on in a retrograde order, which will give the tempered notes of the octaves as well as the former method. By this means the following diviſions for the notes of an octave have been calculated, the length of the whole ſtring having been ſuppoſed equal to 100000.

I.	100000.
✳ b	94387.
II.	89090.
✳ b	84090.
III.	79370.
IV.	74915.
✳ b	70710.
V.	66743.
✳ b	62997.
VI.	59462.
✳ b	56123.

VII. 52973.
VIII. 50000.

If a monochord be divided in this manner, and a harpſichord tuned by it, this inſtrument will then be tuned ſo, that whichever note be taken for the firſt or key-note, its fifth, ſixth, &c. will produce the ſame effect reſpectively.

Thus far I have endeavoured to explain this ſubject in the moſt familiar manner, avoiding as much as poſſible the mathematical language and ſymbols; having found, by experience, that intricate mathematical diſquiſitions, eſpecially on this ſubject, are underſtood only by a few able mathematicians, but that they are neither comprehended, nor even read, by thoſe who might wiſh to underſtand, or to uſe them. It is now neceſſary to conſider this ſubject with reſpect to the practice.

At preſent, the harpſichords and organs are commonly tuned ſo, that ſome concords are very agreeable to the ear, whilſt others are quite intolerable; or, in other words, when the performer plays in certain keys, the harmony is very pleaſing, in others the harmony is juſt tolerable, and in ſome other keys the harmony is quite diſagreeable.

The beſt keys to be played in are the keys of C, of F, of E flat, of B flat, of G and of D in the major mood, and the keys of C, of D, of A, and of B, in the minor mood. Next to thoſe come the leſs agreeable keys of A, A flat, and E in the major mood; beſides thoſe, the reſt are diſagreeable in a greater or leſs degree, ſo that out of twelve keys*, which, on account of the two moods, *viz.* the major and the minor, become twenty-four, there are hardly fourteen that can be uſed; and for this reaſon moſt of the modern compoſitions in muſic are written in thoſe keys.

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* The octave muſt be excepted, in this reſpect it being the ſame as the firſt note.

So far the common method of tuning anſwers ſome purpoſe; for as long as the performer is to play in certain keys only, it is much better to have them tuned in the moſt advantageous manner, than to let thoſe be tuned in a leſs perfect manner for the ſake of others, which he does not intend to uſe. Hence the great harpſichord players generally have their inſtruments tuned in a peculiar manner, *viz.* ſo as to give the moſt advantageous effect to thoſe concords which they more frequently uſe in their compoſitions. And hence alſo, the harpſichords and organs are always tuned different from each other, unleſs they be tuned by the ſame perſon with equal attention, and without any particular inſtructions.

This practice cannot conveniently be laid aſide, *viz.* when the inſtrument is to be tuned for ſolo playing; and for a certain ſtyle of muſic, it is very proper to tune it ſo as to give the greateſt effect to thoſe combinations of ſounds, which are moſtly uſed in thoſe compoſitions. But the caſe is far different when the inſtrument is to ſerve for accompanying other inſtruments in every ſort of muſic, or the voices of good ſingers; for then the diſagreement becomes very audible; and for this purpoſe the harpſichord or organ ought to be tuned according to the above demonſtrated temperament of equal harmony, which is the only one that can poſſibly take place.

When the compoſitions of old maſters are performed in concert, and with the organ or harpſichord tuned in the common manner, the effect is frequently very diſagreeable. This is particularly the caſe with the ſongs of HANDEL, GALLUPPI, LEO, PERGOLESE, and others, who wrote in a great variety of keys, and very often in thoſe, for which the common way of tuning is not at all calculated.

In order to hear the effect of the above-mentioned temperament of equal harmony, I had a monochord made in a very accurate manner, and upon it I laid down the diviſions for the thirteen notes of an octave properly tempered in the manner explained above. After a great deal of trouble in adjuſting the moveable fret, correcting the diviſions, &c. I at laſt ſucceeded ſo well as to render the diviſions exact within at leaſt the 300th part of an inch, and every part of the inſtrument was rendered ſufficiently ſteady and unalterable.

This being done, I had a large harpſichord, with a ſingle uniſon (in order to judge the better of the effect), tuned very accurately by the help of the monochord. With this inſtrument, in whatever key the performer played, the harmony was perfectly equal throughout, and the effect was the ſame as if one played in the key of E natural on a harpſichord tuned in the uſual manner.

I ſhall, therefore, conclude with ſaying, that when the harpſichord, organ, &c. is to ſerve for ſolo playing, and for a particular ſort of muſic, it is proper to tune in the uſual manner, *viz.* ſo as to give the greateſt effect to thoſe concords which occur more frequently in that ſort of muſic; but that when the inſtrument is to ſerve for accompanying other inſtruments or human voices, and eſpecially when modulations and tranſpoſitions are to be practiſed, then it muſt be tuned according to the temperament of equal harmony, which has been explained in the preceding pages.

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